Indian J. Phys. **80** (5), 527-530 (2006)

IJP

A study on the propagation of a Gaussian soliton pulse through an optical fiber when dispersion is compensated accommodating the nonlinearity of the fiber media

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Abstract : It is well known that the propagation of soliton pulse through an optical fiber depends on the compensation of Group Velocity Dispersion (GVD) and Self-Phase Modulation (SPM). This balance is to be maintained exactly where in dispersion compensated fiber, the dispersion are balanced mutually that is waveguide dispersion cancels material dispersion.

In this context, if we consider the nonlinearity in fiber media, then how these dispersions are compensated mutually, is analytically observed by us in this paper. At the same time, how the soliton pulse will be affected by the media of the optical fiber where the dispersion compensation is done accommodating the nonlinearity of the optical fiber.

Keywords : Gaussian soliton pulse, optical fiber, material dispersion, waveguide dispersion, nonlinearity

PACS Nos. : 42.65.Tg, 42.65.Wi, 42.81.Dp

1. Introduction

Optical fibers have created a revolution in the field of telecommunication and have become the back bone of today's global communication networks. Optical fiber communication is considered to be more advantageous because, it has large channel handling capacity, high signal to noise ratio due to presence of low noise, no electromagnetic interference *etc.* In communication engineering, optical fiber and free space communications is being more and more important. Apart from these remarkable advantages, the major disadvantages are information loss and cross talk because of optical loss and dispersion [1]. The problem of dispersion can be overcome with the help of Gaussian soliton pulse [2]. In soliton communication the pulse broadens neither in the time domain nor in frequency domain. In addition to optical communication optical soliton also finds application in pulse compression, photonic switching and soliton laser. It is expected that in future almost all communication will be done by optical soliton. Optical fibers are based on silica glass materials. This material is non crystalline and nonlinear in nature. Control of dispersion effect in optical fiber is a primary requirement. For an ideal communication dispersion compensation fiber is essential. In this paper, we see how dispersion are compensated mutually, if nonlinearity is considered [3–5]. At the same time, how Gaussian soliton pulse will be affected by the media of the optical fiber where the dispersion compensation is done accommodating the nonlinearity of the optical fibre, has also been investigated.

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2. Dispersion in optical fiber

Pulse dispersion represents one of the most important characteristics of an optical fiber. It determines the information-carrying capacity of a fiber optic communication system. In digital communication systems, information to be sent is first coded in the form of pulses and decoded at the receiver after transmission. A pulse of light broadens in time as it propagates through the fiber, this phenomenon is known as pulse dispersion and occurs for three reasons :

(i) Multi-path dispersion, (ii) Material dispersion, (iii) Waveguide dispersion.

2.1. Multi-path dispersion :

When rays making large angles with the axis have to traverse a longer optical path length they take a longer time to reach the output end. So the pulse broadens as it propagates through the fiber. Hence, even though many pulses may be well resolved at the input end, because of broadening of the pulses they may not be so at the out put end. When the out put pulses are not resolvable, no information can be restored. In case of step index fiber, a ray making an angle θ with the axis would occupy a time interval of duration $\Delta T = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right)$ when it traverses a distance L in time t.

Here, (c/n_1) represents the speed of light in the medium of refractive index n_1; c is the speed of light in free space.

2.2. Material dispersion :

Material dispersion is due to the explicit dependence of the refractive index of the core cladding on the wavelength (λ_0). It is a characteristic of the material only. Material dispersion plays a very important role in design of a fiber optics communication system. When a temporal pulse passes through a homogeneous medium, it propagates with a group velocity (v_g) given by

$$1/v_g = dk/dw$$

$$= \frac{d}{dw}\left[\frac{n}{c}n(w)\right],$$

where $k(w) = \frac{w}{c}n(w)$, $n(w)$ = frequency-dependent refractive index.

In terms of wavelength, the group velocity becomes

$$\frac{1}{v_g} = \frac{1}{c}\left[n(\lambda_0) - \lambda_0 \frac{dn}{d\lambda_0}\right].$$

Thus, the time taken by a pulse to traverse length L of fiber is given by

$$\tau = \tau(\lambda_0) = \frac{L}{-} = \frac{L}{-}\ n(\lambda_0) - \lambda_0 \frac{dn}{d\lambda_0}$$

Hence, temporal broadening of the pulse is

$$\nabla\tau = \frac{d\tau}{d\lambda_0}\nabla\lambda_0 = -\frac{L}{c}\left(\lambda_0^2 \frac{d^2 n}{d\lambda_0^2}\right)\left(\frac{\nabla\lambda_0}{\lambda_0}\right).$$

The above broadening is referred to as material dispersion. Material dispersion in terms of ps/km-nm is

$$D_m = -\frac{1}{\lambda_0 c}\left(\lambda_0^2 \frac{d^2 n}{d\lambda_0^2}\right)\left(\frac{\nabla\lambda_0}{\lambda_0}\right)10^9 \text{ ps/km-nm},$$

where λ_0 is measured in micro meters and $c = 3 \times 10^5$ km/s.

For pure silica, at $\lambda_0 = 1550$ nm, $\frac{d^2 n}{d\lambda_0^2} = -4.2\times10^{-3}\mu\text{m}^{-2}$, so that $D_m \approx 22$ ps/km-nm.

2.3. Waveguide dispersion :

Wave guide dispersion is due to the explicit dependence of the refractive index of the core and cladding on the frequency (w). In case of wave guide dispersion, wave propagation vector

$$\beta = \frac{w}{-}\left[n_2 + (n_1 - n_2)\, b(v)\right],$$

where b–normalized propagation constant, $v = \frac{w}{c}a\sqrt{n_1^2 - n_2^2}$, the waveguide parameter, a is the radius of the core, n_1 and n_2 are the core and cladding refractive index.

Hence, group velocity is given by

$$\frac{1}{v_g} = \frac{d\beta}{dw} = \frac{n_2}{c} + \frac{n_1 - n_2}{c}\left[\frac{d}{dv}(bv)\right].$$

Thus, the time taken by a pulse to traverse length L of the fiber is given by

$$\tau_w = \frac{L}{v_g} = \frac{L}{c}n_2\left[1 + \Delta\frac{d}{dv}(bv)\right],$$

where $\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \cong \frac{n_1 - n_2}{n_2}$.

For a source having a spectral width $\Delta\lambda_0$, the corresponding waveguide dispersion is given by

$$\Delta\tau_w = \frac{d\tau}{d\lambda}\Delta\lambda_0 \approx \frac{L}{c}n_2\Delta\left(\frac{\Delta\lambda_0}{\lambda_0}\right)\left[v\frac{d^2(bv)}{dv^2}\right].$$

In terms of ps/km-nm, the waveguide dispersion is given by

$$D_w = -\frac{n_2\Delta}{c\lambda_0}\left(v\frac{d^2(bv)}{dv^2}\right)10^7 \text{ ps/km.nm},$$

where λ_0 is measured in nanometer. For step index fiber,

$$\frac{d^2(bv)}{dv^2} = \frac{2B^2}{v^2}.$$

In case of pure silica, at λ_0 = 1550 nm, B = 0.996, a = 2 μm, n_1 = 1.46, n_2 = 1.45, so that $D_w \approx 22$ ps/km-nm.

3. Dispersion compensation fiber

Dispersion compensating fiber is a fiber that balances the pulse broadening mutually. Total dispersion of a signal mode fiber is given by

$$D_t = D_m + D_w = -\frac{2\pi c}{\lambda_0^2}\frac{d^2\beta}{dw^2}.$$

How this dispersion is compensated mutually? Let us consider a pulse propagating through an optical fiber. We know group velocity attains a maximum value at the zero dispersion wavelengths and on either side it is monotonically decreases with wavelength. Then the red components of the pulse will travel slower than the blue components of the pulse. Because of this fact, the pulse will get broadened. The leading edge of the output pulse is blue shifted and the trailing edge is red shifted. Then after propagating through such a fiber for a certain length L_1. We allow the pulse to propagate through another fiber, where the red components will now travel faster than the blue components and the pulse will tends to reshape itself into its original form. This is the basic principle behind dispersion compensation. That is, if $(D_t)_1$ and $(D_t)_2$ be the dispersion coefficients of the first and second fibers and if the length of the two fibers (L_1 and L_2) are such that

$$(D_t)_1\, L_1 + (D_t)_2\, L_2 = 0,$$

then the pulse emanating from the second fiber will be same to the pulse entering the first fiber.

4. Effect of nonlinearity in dispersion compensation fiber

Here, we treated the optical fiber as a nonlinear medium, that is intensity dependent refractive index. Consider the propagation of an optical pulse in a lossy fiber characterized by the refractive index

$$n(w) = n_0(w) + n_{20}I,$$

where $n_0(w)$ is the linear part of the refracting index, $n_{20}I$ is the intensity-dependent nonlinear part.

In case of material dispersion, if we use nonlinear effect, the group velocity (v_g) is given by

$$\frac{1}{v_g} = \frac{dk}{dw} = \frac{d}{dw}\left[\frac{w}{c}n(w)\right],$$

where $k = \frac{w}{c}n(w)$.

$$\frac{1}{v_g} = \frac{d}{dw}\left[\frac{w}{c}\{n_0(w) + n_{20}I\}\right],$$

$$\frac{1}{v_g} = \frac{1}{c}\left[n_0(w) + n_{20}I + w\left(\frac{dn_0}{dw} + I\frac{dn_{20}}{dw}\right)\right].$$

Now, group velocity in terms of the free space wavelength $\lambda_0\ (= 2\pi c/w)$ is given by

$$\frac{1}{v_g} = \frac{1}{c}\left| n_0(\lambda_0) + n_{20}I - \lambda_0\frac{dn_0}{d\lambda_0} - \lambda_0 I\frac{dn_{20}}{d\lambda_0}\right.$$

Hence, the time taken by the pulse to traverse length L of the fiber is given by

$$\tau = \tau(\lambda_0) = \frac{L}{v_g}\left[n_0(\lambda_0) + n_{20}I - \lambda_0\frac{dn_0}{d\lambda_0} - \lambda_0 I\frac{dn_{20}}{d\lambda_0}\right].$$

If the spectral width of the source is $\nabla\lambda_0$, the temporal broadening of the pulse is given by

$$\nabla\tau = \frac{d\tau}{d\lambda_0}\nabla\lambda_0 = -\frac{L}{c}\ \lambda_0\frac{d^2n_0}{d\lambda_0^2} + \lambda_0 I\frac{d^2n_{20}}{d\lambda_0^2}$$

$$\lambda_0^2\left\{\frac{d^2n_0}{d\lambda_0^2} + \frac{d^2n_{20}}{d\lambda_0^2}I\right\}\left(\frac{\nabla\lambda_0}{\lambda_0}\right.$$

We know $n_{20} = \dfrac{3\chi^3}{4c\epsilon_0 n_0^2}$,

$$\frac{d^2n_{20}}{d\lambda_0^2} = \frac{9\chi^3}{2c\epsilon_0 n_0^4}\frac{d^2n_0}{d\lambda_0^2} = \frac{6n_{20}}{n_0^2}$$

Thus, $\nabla\tau = -\frac{L}{c}\ \frac{d^2n_0}{d\lambda_0^2}\ 1 + \frac{6n_{20}}{n_0^2}I\Big\}\Big]\left(\frac{\nabla\lambda_0}{\lambda_0}\right)$,

$$D_{mn} = -\frac{10^9}{\lambda_0 c}\left(\lambda_0^2\frac{d^2n_0}{d\lambda_0^2}\quad 1 + \frac{6n_{20}}{n_0^2}I\right. \text{ ps/km-nm.} \tag{i}$$

In case of waveguide dispersion, we consider that v (waveguide parameter) is approximately constant. Then

$$D_{wn} = -\frac{n_2}{n_2}\left(n_0 + n_{20}I - n_2\right)\frac{10^7}{3\lambda_0}\left(\frac{d^2(bv)}{dv^2}\right),$$

$$D_{wn} = -\frac{n_2\Delta}{c\lambda_0}\left(v\frac{d^2(bv)}{dv^2}\right)10^7\left(1+\frac{n_{20}I}{\Delta}\right) \text{ ps/km-nm, (ii)}$$

where $\Delta = \dfrac{n_0 - n_2}{n_2}$.

5. Conclussion

Here, we conclude that when nonlinearity is absent, material dispersion and waveguide dispersion are mutually cancelled at λ_0 = 1550 nm. This wavelength is called zero dispersion wavelength. Hence, we can infer that due to nonlinearity material dispersion increases by a factor $\left(1+ 6n_{20}I\right.$ and waveguide dispersion increases by a factor $\left(1+\frac{n_{20}I}{\Delta}\right)$. To compensate large material dispersion, waveguide dispersion should be equally increased. We can control wave-guide dispersion by changing Δ and the radius of the core. Again, if the intensity term within the expression (i) and (ii) is controlled, we can change the broadening of the soliton pulse. This is the most important finding in the above treatment.

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